Mathematical modelling of thermal performance using transient heat transfer analysis of solar flat plate collector

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Abstract

The study presents a one-dimensional mathematical model using general energy balance equation for simulation of transient heat transfer on different nodes by considering their capacity and heat loss effect on solar flat-plate collector (FPC). The proposed model considers the distributed factors of collector components. Theoretical analysis is carried out for understanding and obtaining factors affecting thermal performance of FPC. The governing equations describe the net heat transfer for glass cover, air gap between absorber and cover, absorber, working fluid and insulation. The differential equations are solved using the implicit finite-difference method by performing iterations by numerical method.

Keywords: solar flat-plate collector mathematical model, parameters, transient analysis

1. Introduction

Fossil fuel plays crucial role in the world energy market to produce electricity. Electricity still has major stake for space heating and water heating. As fossil fuels are non-renewable source of energy, it's only a matter of time when it gets over. This is evident from various studies of different models; one such model predicts depletion time for fossil fuels like oil (35 years), coal (107 years) and Gas (37 years) [1]. Fossil fuels have environmental impacts such as Global warming, Greenhouse effect, Ozone layer depletion and so forth. So, to overcome environmental issues, renewable resources are considered as clean sources of energy and optimal use of these resources minimize environmental impacts, produce minimum secondary wastes and are sustainable based on current and future economic and social societal needs. Within renewable energy sources, solar energy standout amongst the peer because of its vast application and easy availability. India is endowed with vast solar energy potential about 5000 kWh per year is incident over India's land area with most parts receiving 4-7 kWh per sq. meter per day about 325 days [2].

Solar radiation can be converted into heat for thermal application by solar collector such as Flat plate collector. Solar flat plate collectors are devices used to trap solar thermal energy and use it for heating applications like water heating, room heating and other industrial applications. Flat-plate collector, developed by Hottel and Whillier in the 1950s, was the most common type. They consisted of (1) a dark flat-plate absorber, (2) a transparent cover that reduces heat losses, (3) a heat-transport fluid (air, antifreeze or water) to remove heat from the absorber, and (4) a heat insulating backing. Over the period of time many modifications are done to enhance its efficiency and parameter optimization. To get optimum collection of solar radiation at moderate temperatures is one of the most challenging problem, so by comparing three transparent plastic films like 1) Teflon 2) Weatherable Mylar 3) Polyethylene with glass as glazing for solar energy absorption collectors was studied and the results showed that Teflon have the best transmission capacity amongst all three with different altitude and tilt angle [3]. The other important aspect for enhancing thermal performance is to get maximum radiation on collector, in doing so arrangement of side mirrors on FPC was carried on. The addition of side mirrors can increase the output of fixed FPC and permit higher working temperatures [4]. The problem of the high heat loss from cover plate was addressed by using low density Styrocel as the transparent cover. Optimum thickness of Styrocel layer was found to be between (0.5-1) inches. [5]. Simulation study on the influence of cover design on thermal performance of FPC enlightens that single high transmittance cover of FEP Teflon or low iron glass gives better thermal performance over multiple cover plates [6]. Oxides such as indium oxide and aluminium doped zinc oxide were used for glass cover coating on single and double glass glazed and analysed experimentally. The results showed that a significant increase in efficiency is accessible both in single-glazed collectors with low or non-selective glass cover and in double-glazed collectors with highly selective glass cover [7].

After glass cover, next essential aspect is air gap for which experimental work to increase transmissivity of air gap and minimize the emissivity of absorber by installing the honeycomb panel structure of various thicknesses between glass cover and absorber was undertaken [8]. Further, shade correction factor was introduced and evaluated free convection heat transfer coefficient for different collector orientation was carried out and results showed that optimum air gape should be 4-5 cm to reduce shadow cast [9]. The use of slit honeycomb for suppressing natural convection heat loss and also doing minor modification of honeycomb structure with polycarbonate sheet of 16 mm increased the transmissivity and reduces top heat loss [10, 11]. Apart from introducing new material, water vapor was used as replacement of air and efficiency of FPC increased by increasing transmittance of heat and heat loss also reduced [12].

Next crucial component in the story is absorber, to achieve high absorptivity for solar radiation and low emissivity, coatings of CuO and Co_3O_4 on polished nickel, silver, and platinum were applied. Co_3O_4 coating on silver gave best performance [13]. Experimental work was also under taken to enhance the thermal performance of FPC by indenting on absorber surface to increase overall surface area [14]. The importance of pitch distance in selecting an absorber plate having high value of conduction heat transfer rate per unit temperature was revealed by experimental study on transient thermal behaviour of wickless pipe [15]. The effect of colour on the thermal performance of absorber was examined both theoretically and experimentally for colours ranging from white to black, with black colour on absorber showed optimum efficiency [16]. Further, experimental comparison was carried out between nickel-tin coated absorber and black painted absorber which deduced that the prior one was 29.23% more efficient than the latter [17].

Working fluid is another important component of FPC, which is responsible of carrying heat from absorber plate. Many experimental as well as theoretical studies have been undertaken to study properties of different fluids and getting better heat exchange. One such experimental study was performed to investigate the effect of Cu nanoparticle in ethylene glycol as a solvent with different mass flow rate and concentration on the efficiency [18]. Further Propylene glycol was replaced by ethylene glycol as a working fluid and results indicated that the thermal performance of FPC increases by a considerable amount [19]. Silver-water nanofluid was used as alternative working fluid. Experimental results were compared with ANN (Artificial neural network) in MATLAB and both converged to same conclusion [20].

Apart from enhancing the component efficiency, a study focusing on other factors like weather parameters, average diffuse radiation, cloudiness and so forth was undertaken [21]. Important heat loss coefficient was derived theoretically and was evaluated experimentally, which improved thermal efficiency[22]. Theoretical work was carried out by using various mathematical model for evaluating the fluid temperature distribution along the flat plate solar collector and calculating useful heat gain and optimization of tilt angle under actual dynamic conditions [23,24]. Analysis of dynamic behaviour of FPC was done using one node model mathematical model, computational fluid dynamics (CFD) and lumped capacitance model and was compared to establish standards on ASHRAE 93-86 standard and British standard BS 6757 [25,26,27]. Further carrying out modifications basic design, fin geometry is changed to enhance heat removal. Triangular fins were attached to absorber and experimental conclusion was drawn that heat-removal factor for each triangular fin is greater than rectangular fin of the same base width [28,29].

The next frontier was followed by change in basic design of FPC. Instead of insulation layer at the bottom, reflecting surface was used to incident radiation from bottom as well, which resulted in higher fluid temperatures [30].

2. Methodology

In real situation in flat plate collector temperature of fluid changes at any instant of time with space coordinate. To analyze performance of flat plate collector, we need to know temperature at each time. So, temperature is varied with time as well. Transient heat transfer analysis has been carried out by numerical method.

2.1 Governing Equation

Transient energy balance for all components can be written as.

1.Absorber

$$\left[I(\tau\alpha) - h_r\left(T_{ab} - T_g\right) - h_{ag}\left(T_{ab} - T_{ag}\right) - \frac{K_i}{\delta_i}(T_{ab} - T_i)\right]\Delta z * b - \pi d_{in}\Delta z h_f\left(T_{ab} - T_f\right) = c_{ab}\rho_{ab}V_{ab}\frac{dT_{ab}}{d\tau} \quad (1)$$

2. Glass cover

$$\left[I(\alpha) - h_{atm}\left(T_g - T_{atm}\right) - h_r\left(T_g - T_{ab}\right) - h_{ag}\left(T_g - T_{ag}\right)\right]\Delta z * b = c_g\rho_g V_g\frac{dT_g}{d\tau} \quad (2)$$

3. Air gap

$$\left[h_{ag}\left(T_g - T_{ag}\right) + h_{ag}\left(T_{ab} - T_{ag}\right)\right]\Delta z * b = c_{ag}\rho_{ag}V_{ag}\frac{dT_{ag}}{d\tau} \quad (3)$$

4.insulator

$$\left[\frac{K_i}{\delta_i}(T_{ab} - T_i) - h_{atm}(T_i - T_{atm})\Delta z * b\right] = c_i\rho_i V_i\frac{dT_i}{d\tau} \quad (4)$$

5.Working fluid

$$\left[\pi d_{in}h_f\left(T_{ab} - T_f\right) - \dot{m}_f c_f\frac{\partial T_f}{\partial z}\right] = c_f\rho_f V_f\frac{\partial T_f}{\partial \tau} \quad (5)$$

2.2 Method for solving transient equations

The governing equations (1 through 5) are solved by finite difference method. The solution was then advanced from present to future by using small increment of time d**τ**. The accuracy of is inversely proportional to time step. The length of collector is divided into number of nodes to obtain temperature at different nodes. The equation in finite difference form is written as follow.

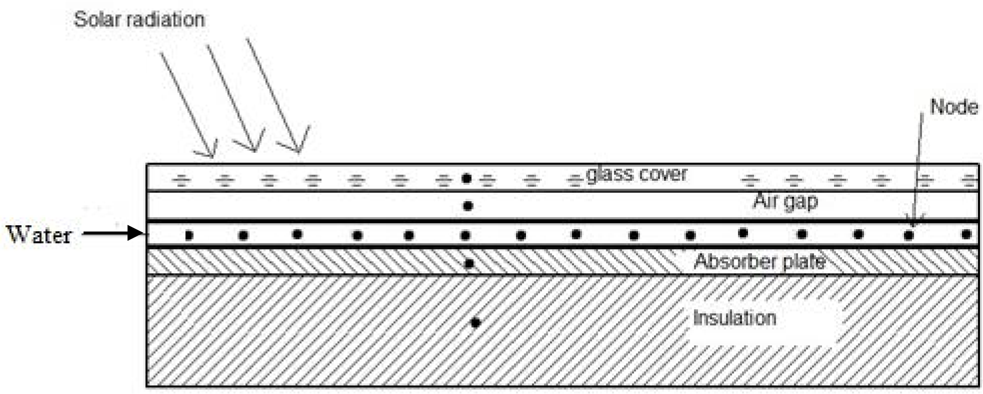

Figure 1 Schematic diagram of FPC with nodes

Diagram depicts the nodal distribution along the direction of fluid flow (j-nodes) and along the components (5 nodes) to measure change in temperature with time.

$$\frac{dT_g}{d\tau} = \frac{T_{g,j}^{\tau+\Delta\tau} - T_{g,j}^{\tau}}{\Delta\tau}, \frac{dT_{ab}}{d\tau} = \frac{T_{ab,j}^{\tau+\Delta\tau} - T_{ab,j}^{\tau}}{\Delta\tau}$$
$$\frac{dT_i}{d\tau} = \frac{T_{i,j}^{\tau+\Delta\tau} - T_{i,j}^{\tau}}{\Delta\tau}, \frac{\partial T_f}{\partial\tau} = \frac{T_{f,j}^{\tau+\Delta\tau} - T_{f,j}^{\tau}}{\Delta\tau} \quad (6)$$
$$\frac{\partial T_f}{\partial z} = \frac{T_{f,j}^{\tau+\Delta\tau} - T_{f,j-1}^{\tau+\Delta\tau}}{\Delta z} \quad (7)$$

Time derivatives $\frac{dT}{d\tau}$ in governing equations (1 through 4) are substituted by forward difference equation (6) and dimensional derivatives $\frac{dT}{dz}$ in governing equation (5) is substituted by backward difference equation (7). Further transformations are carried out and following equation were obtained.

$$T_{g,j}^{\tau+\Delta\tau} = \frac{1}{F_j\,\Delta\tau} T_{g,j}^{\tau} + \frac{B_j}{F_j} T_{atm}^{\tau+\Delta\tau} + \frac{C_j}{F_j} T_{ab,j}^{\tau+\Delta\tau} + \frac{D_j}{F_j} T_{ag,j}^{\tau+\Delta\tau} + \frac{E}{F_j} I^{\tau+\Delta\tau} \quad (8)$$

$$T_{ag,j}^{\tau+\Delta\tau} = \frac{1}{H_j\,\Delta\tau} T_{ag,j}^{\tau} + \frac{G_j}{F_j}(T_{g,j}^{\tau+\Delta\tau} + T_{ab,j}^{\tau+\Delta\tau}) \quad (9)$$

$$T_{ab,j}^{\tau+\Delta\tau} = \frac{1}{Q_j\,\Delta\tau} T_{ab,j}^{\tau} + \frac{L_j}{Q_j} T_{g,j}^{\tau+\Delta\tau} + \frac{M_j}{Q_j} T_{a,j}^{\tau+\Delta\tau} + \frac{O_j}{Q_j} T_{f,j}^{\tau+\Delta\tau} + \frac{K_j}{Q_j} I^{\tau+\Delta\tau} + \frac{P_j}{Q_j} T_{i,j}^{\tau+\Delta\tau} \quad (10)$$

$$T_{f,j}^{\tau+\Delta\tau} = \frac{1}{U_j\,\Delta\tau} T_{f,j}^{\tau} + \frac{R_j}{U_j} T_{ab,j}^{\tau+\Delta\tau} + \frac{S_j}{U_j \Delta z} T_{f,j-1}^{\tau+\Delta\tau} \quad (11)$$

$$T_{i,j}^{\tau+\Delta\tau} = \frac{1}{X_j\,\Delta\tau} T_{i,j}^{\tau} + \frac{V_j}{X_j} T_{ab,j}^{\tau+\Delta\tau} + \frac{W_j}{X_j} T_{atm}^{\tau+\Delta\tau} \quad (12)$$

Constants in equations (8) to (12) are

$$B_j = \frac{h_{atm,j}}{c_g \rho_g \delta_g}, C_j = \frac{h_{r,j}}{c_g \rho_g \delta_g}, D_j = \frac{h_{g,j}}{c_g \rho_g \delta_g}, E = \frac{\alpha}{c_g \rho_g \delta_g}, F_j = \frac{1}{\Delta\tau} + D_j + C_j + B_j, K_j \frac{b(\tau\alpha)}{J_j}, L_j = \frac{b h_{r,j}}{J_j}$$

$$, M_j = \frac{b h_{g,j}}{J_j}, O_j = \frac{\pi d_{in} h_{f,j}}{J_j}, P_j = \frac{b k_i}{J_j \delta_i}, G_j = \frac{h_{g,j} b}{c_{ag} \rho_{ag} (b\delta_{ag} - \pi r_o^2)}$$

$$, H_j = \frac{1}{\Delta\tau} + 2G_j, , S_j = \frac{\dot{m}_f}{\rho_f A_c}$$

$$J_j = c_{ab} \rho_{ab} [b\delta_{ab}\, \pi(r_o^2 - r_{in}^2)] ,$$

$$Q_j = \frac{1}{\Delta\tau} + L_j + M_j + O_j + P_j \quad ,R_j = \frac{\pi d_{in} h_{f,j}}{c_f \rho_f A_c}$$

$$U_j \frac{1}{\Delta\tau} R_j + \frac{S_j}{\Delta z} \;, V = \frac{2k_i}{c_i \rho_i \delta_i^2} \;, \quad W_j = \frac{2h_{atm,j}}{c_i \rho_i \delta_i} \quad , X_j = \frac{1}{\Delta\tau} + V + W_j$$

Currently evaluated temperature should satisfy the below condition and if it doesn't follow then Iteration will takes place

$$\frac{\left|Y_{j,(k+1)}^{\tau+\Delta\tau} - Y_{j,(k)}^{\tau+\Delta\tau}\right|}{Y_{j,(k+1)}^{\tau+\Delta\tau}} \leqslant \mathbf{10^{-6}} \quad (13)$$

To determine heat transfer co efficient of working fluid, air gap and atmosphere

$$h = \frac{Nu\, k}{\delta} (14)$$

Empirical Heaton formula

$$Nu_{f,j} = Nu_\infty + \frac{a(Re_{f,j} Pr_{f,j} (d_{in}/L))^k}{1 + b(Re_{f,j} Pr_{f,j} (d_{in}/L))^n} (15)$$

$$1 < Re_{f,j} Pr_{f,j}\, d_{in}/L \leqslant 1000$$

Radiative heat transfer coefficient,

$$h_{r,j} = \frac{\sigma (T_{ab,j}^2 - T_{g,j}^2)(T_{ab,j} - T_{g,j})}{\left(\frac{1}{\varepsilon_{ab}}\right) + \left(\frac{1}{\varepsilon_g}\right) - 1} \quad (16)$$

Empirical correlation was given by Hollands formula

$$Nu_{a,j} = 1 + 1.44\left\{1 - \frac{1708[\sin(1.8\beta)]^{1.6}}{Ra_j \cos\beta}\right\} \times \left(1 - \frac{1708}{Ra_j \cos\beta}\right)^+ + \left[\left(\frac{Ra_j \cos\beta}{5830}\right)^{\frac{1}{3}} - 1\right]^+ \quad (17)$$

$$Nu_{am} = 0.86 Re_{am}^{\frac{1}{2}} Pr_{am}^{\frac{1}{3}} \text{and} \delta = \frac{4ab}{\sqrt{a^2 + b^2}} (18)$$

$$h_{r,j} = \frac{\sigma \varepsilon_g \left(T_{ab,j}^4 - T_{sky}^4\right)}{T_{g,j} + T_{atm}} + h_g \quad (19)$$

$$T_{sky} = 0.0552 T_{am}^{1.5} (20)$$

$$h_{i,am,j} = \frac{\sigma \varepsilon_i \left(T_{i,j}^4 - T_{sky}^4\right)}{T_{i,j} + T_{am}} + h_{c2} (21)$$

3. Result & Discussion

Figure 1 shows the experimental and the theoretical set up of the system. Figure 2 shows the experimental values for irradiation. For this irradiation, the measured values of heated water are shown in figure 3. For comparison, figure 3 also shows the calculated water temperature. It shows a good resemblance of the measured and calculated values. The maximum temperature is reached at 4 pm in the day when there is a maximum heating of the absorber plate occurs. This maximum heating of the absorber plate occurs just after the maximum radiation for the day as seen from figure 2. This is because the transient nature of the heat absorption in to the absorber plate. Figure 4 shows the comparison of calculated water temperature with and without glass cover. It can be observed that the maximum temperature attained with glass cover is lower. This is because the thermal resistance offered by glass cover.

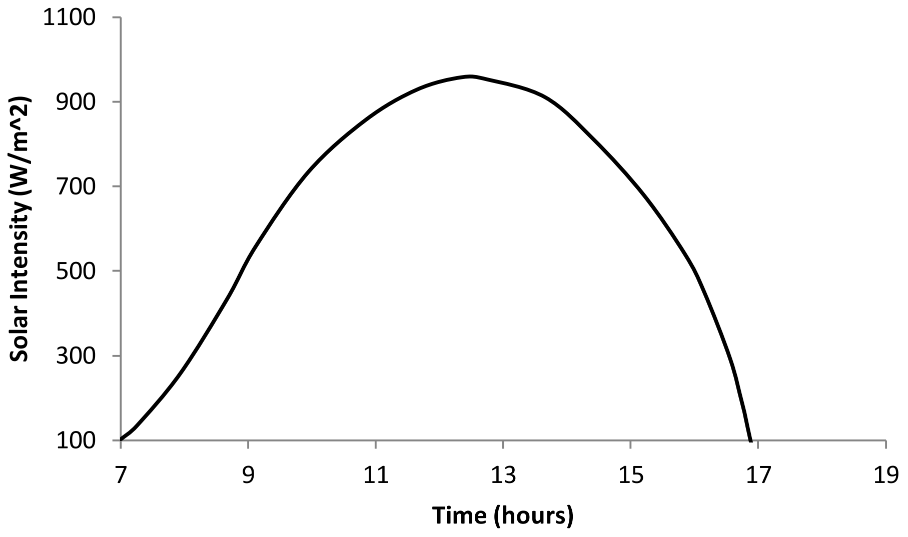

Figure 2 Measured solar intensity

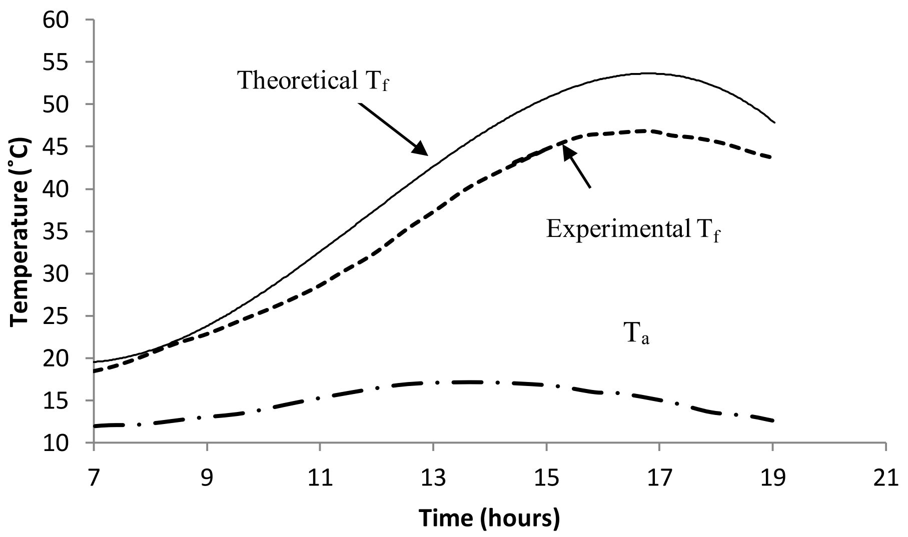

Figure 3 Comparison of theoretical and experimental temperature

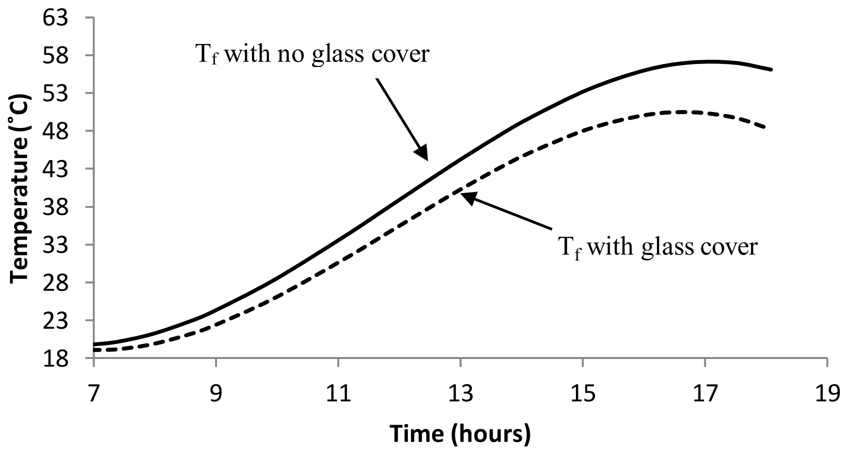

Figure 4 Effect of glass cover on water temperature

4. Conclusion

A good theoretical model has been developed which is capable of predicting the performance of solar water heater. This model concludes that the glass cover reduces the water temperature, however this is observed only during the day time and further studies are required to conclude about the effect of glass cover during the night hours.

Nomenclature

I = Incidence solar radiation, W/m^2
τ = Transmissivity
α = Absorptivity
h_r= Radiative heat transfer co-efficient, W/m^{2}K
h = Heat transfer co-efficient, W/m^{2}K
Nu = Nusselt number
Re = Reynolds number
Pr = Prandtl number
Ra = Rayleigh number
σ = Stefan-Boltzmann constant, W/m^2k^4
T= Temperature, K
K= Thermal conductivity, W/m-K
δ= Thickness, m
d_{in}= Inner diameter of absorber tube, m
c= Specific heat, KJ/kg-K
ρ= Density, kg/m^3
V= Volume, m^3
Δz= Length of control volume, m
b = width of collector, m
A_c = Cross-sectional area of absorber pipe, m^2
$\dot{m}$= Mass flow rate, kg/sec
ab = absorber
ag = air gap
atm = atmosphere
g = glass cover
i = insulator
f = fluid
j = nodes

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