On Construction of Golden Section Octagon

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Received: 26 June 2017, Revised: 27 August 2019, Accepted: 28 August 2019, Published: 25 September 2019

Abstract

The present author defines the golden section octagon as the eight – sided closed figure for which the ratio of the two adjacent sides (longer to shorter side) gives the value of golden number φ = 1.6180, unlike in regular octagon for which the ratio of the two adjacent sides gives a number 1. The few common shapes which involves golden ratio in its geometry are rectangle, triangle and pentagon. It is known that such shapes appear in all sorts of discipline, science, technology, art, architecture and in nature. Though the present work inspires from the shapes which are in existence, exact occurrence of golden section octagon in nature and other disciplines is not known. The main objective of this paper is to explore the method for geometrical construction of golden section octagon and study its property using the concept of golden ratio and golden sections.

Keywords

Golden section, golden octagon, golden ratio, Fibonacci Sequences

Introduction

Golden ratio is a special irrational number found by dividing a line into two parts such that the longer part divided by the smaller part of the line is also equal to the total length divided by the longer part of the line. The division of the parts of the lines has the approximate value equal to 1.6180. It is denoted by the Greek letter φ[1, 2].

This concept can be understood further by dividing the line in two parts as shown below. In this line, the line has two parts, AC and CB. Here, AC is larger part with reference to CB and also AB is larger part as compared to AC. These two parts can be in golden ratio if their ratio AC/CB and AB/AC gives 1.6180. There is an occurrence of various polygons whose adjacent sides are in golden ratio if the ratio is approximately equal to 1.6180 can be called as golden section polygon. Such golden section polygons are rectangles, triangles and pentagon. More details of these shapes having golden section properties are discussed in the literature[3, 4]. In the present work, one of such golden section polygon, octagon having golden section properties is evolved and investigated which is unexplored elsewhere. The main objective of this construction method is (1) to obtain the general construction method which is applicable to any dimensions and any scale (n_1/n_2 = 1.6180) rather than specific cases such as two adjacent side of polygon has to be 8/5 = 1.6180, 13/8 = 1.6180 etc… and is achieved by measuring and drawing it. To make the method more valid for any general construction and geometrical parameters (2) To explore the geometrical properties new shape of golden section octagon (3) to enable the designers to use the shapes in the various designs.

$$\frac{AB}{AC}=\frac{AC}{CB}=\varphi\cong 1.6180 \tag{1}$$

Geometry and Construction

Construct an equilateral triangle ABC as shown in Fig. 1. Each sides of the triangle are named BA, AC and CB. On each of these line segments, obtain the points G, H, I, D, E, F in such a way (Procedure is given in Appendix I) that following property of the ratios on each of the line segments hold true for the constructed equilateral triangle.

$$\begin{gathered}\frac{AB}{BG}=\frac{AB}{AD}=\frac{AD}{AG}=\frac{BG}{BD}=\frac{AG}{GD}=\frac{BD}{DG}=\varphi\\ \frac{AC}{AH}=\frac{AC}{CE}=\frac{AH}{AE}=\frac{CE}{CH}=\frac{AE}{EH}=\frac{CH}{HE}=\varphi\\ \frac{BC}{BF}=\frac{BC}{CI}=\frac{BF}{FC}=\frac{CI}{CF}=\frac{BI}{IF}=\frac{CF}{FI}=\varphi\end{gathered} \tag{2}$$

Join line GE, DI, HF to complete the inside polygon GEHFIDG, makes golden section hexagon for which following holds true.

$$\frac{GE}{EH}=\frac{HF}{FI}=\frac{ID}{DG}=\varphi \tag{3}$$

The complete proof of Eq. 3 is given in[5].

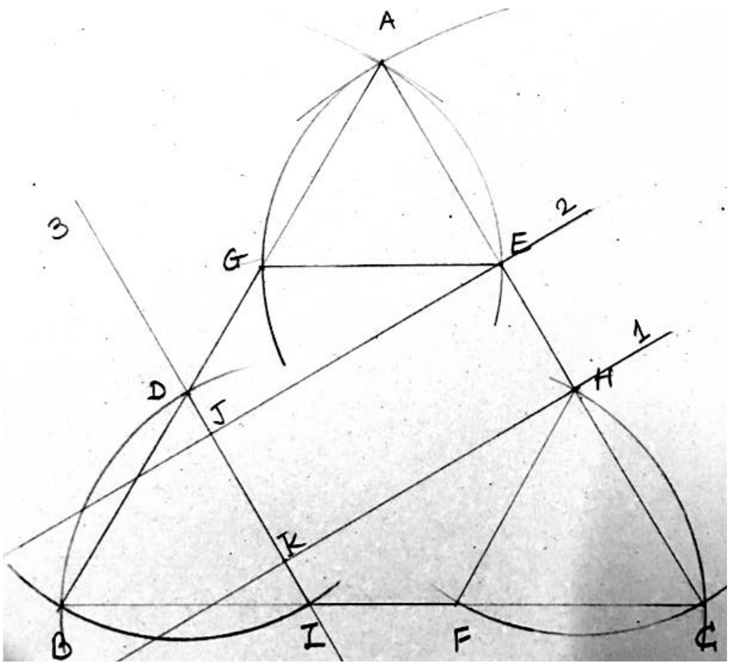

Figure 1: Golden Section on equilatral triangle

Take length GE and draw an arc from E of length GE passing through G and A. Repeat the similar procedure to draw the remaining arc seen in Fig. 1, as per the following Table 1 .

Sr. No.	Arc Length	From point	Points passing through	Covering Length
1	GE	G	E and A	AE
2	FH	H	C and F	CF
3	FH	F	C and H	CH
4	DI	D	B and I	BI
5	DI	I	B and D	BD

Table 1: Points for the Arc passing through A, B and C

For second construction phase, draw two parallel lines 1 and 2 passing through points E, J and H, K respectively. These lines are being noted as EJ and HK. By drawing the line, it is ensured that EH = JK. Draw a line passing through DJKI – line number 3, which also ensures DJ = KI.

Draw the perpendicular line from point M marked on the line 3. Mark M as the origin of this line to the point on the arc AG marked as L, which has the similar length as EH. EH = ML as shown in Fig. Line ML should remain parallel to lines 1 and 2. Repeat the process similar on the mirror side of the point M marked as N. Draw perpendicular line passing through N and projecting on the arc FC in such a way that ML = NO' = EH.

Bisect newly formed angles GEL and FHO'. Mark new bisection points on arc GL and FO' as P and Q, respectively. Bisecting angle procedure is performed to achieve the required angle in octagon. Now mark new points on arc AG and CF as R and S respectively in such a way that it has the arc length as PL = LR and QO' = O'S, respectively (Figure 2).

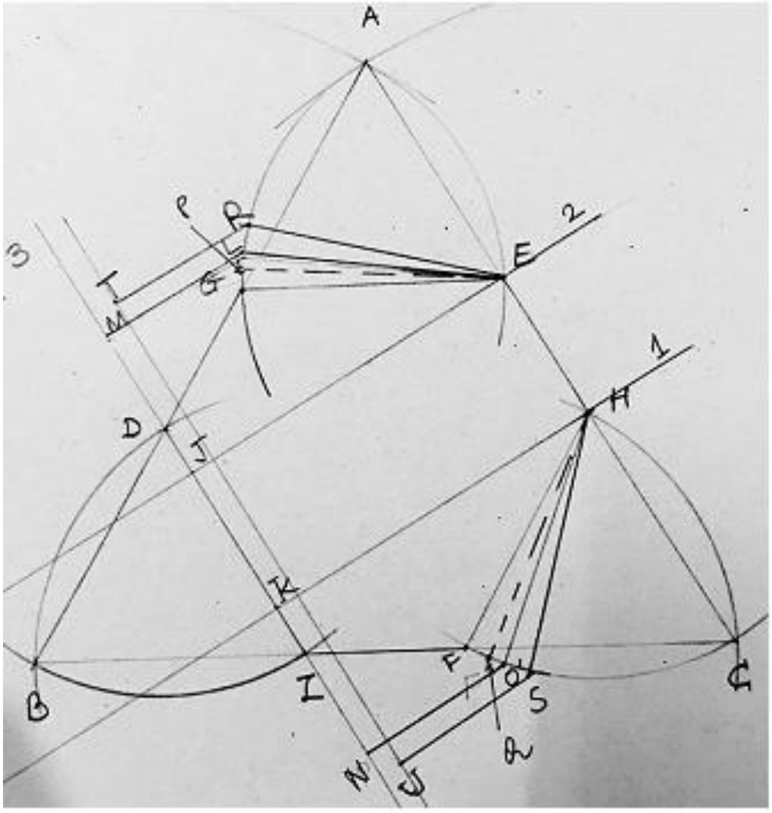

Figure 2: Constructing Golden Octagon

From R and S, draw parallel line such a way that it is parallel to the lines ML and NO’. Name these lines as RT and SU. Again, Points T and U are obtained in such a way so that it gives TR = EH and US = EH. Draw one line passing through T and U. Draw arcs named as 4 and 5 from T and U such as it has the length ER and HS, which also interacts line 1 and 2 respectively.

Intersection of the arc 4, arc BD and line 2 meet at the point V, similarly intersection of an arc 5, arc BI and line 1 meet at point W. Join VW. Also, join TV and UW. Complete ERTVWUSH, is golden section octagon as seen in Fig. 3. Some geometrical properties of golden section Octagon is observed and written as follows. VT/TR = φ, RE/EH = φ, HS/SU= φ, WU/WV = φ.

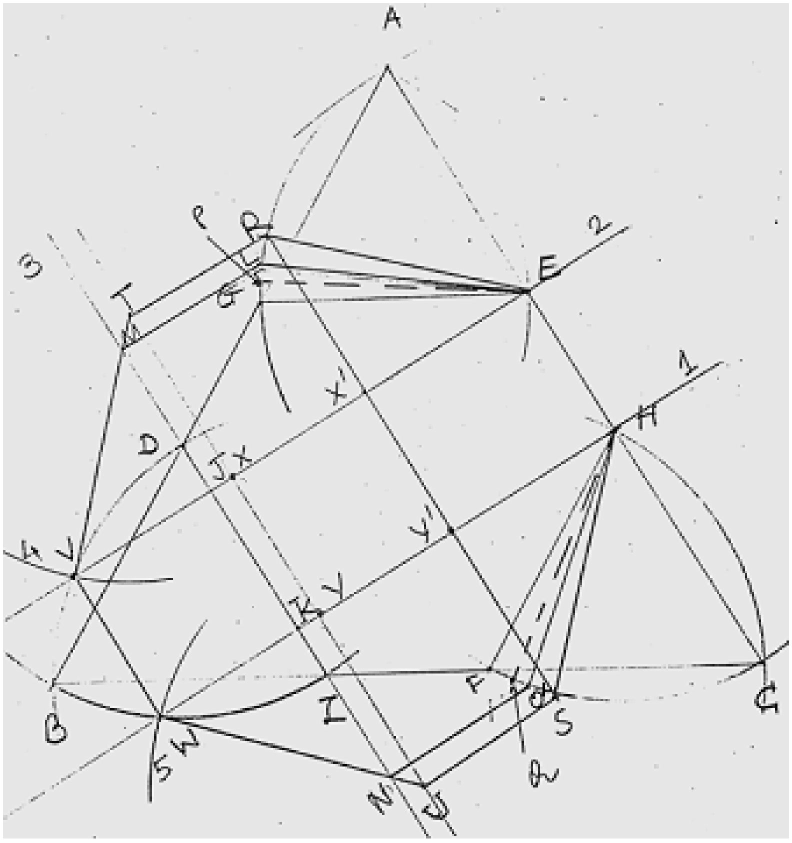

Figure 3: Golden Section Octagon

Properties of Constructed Golden Octagon

Constructed octagon is irregular, as the ratio of the longer side to shorter side should give golden number φ, since all the sides are incongruent. They also have incongruent angles, which mean that the measurement of all the angles of the triangles inside golden octagon is also incongruent. However, exterior angle of golden octagon is 135°, which the case for regular octagon is.

As shown in Fig. 4. , various interior angles of the octagon such as $\angle$VTR, $\angle$ERT, $\angle$TVW, $\angle$UWV, $\angle$USH and $\angle$HER gives 135°. The triangles are obtained by joining the opposite points and lines as shown in Fig. 4. The interior angles of the triangles are 34° and 56° for two incongruent triangles which are congruent alternatively. The ratio of these angles gives the value φ. The remaining angles for this triangles are 73° and 62°. (Fig. 4). For example, in fig. 4, these triangles are: OUS, OEH, OTR and OWV are congruent and have interior angle of 34°. Remaining two angles for this triangle is 73°. OER, OHS, OUW, OTV are congruent with each other and interior angle is 56°. Remaining two angles for these triangles are 62°. Here notation O is the central origin of the octagon.

The area of the Golden section octagon is given as below:

Assume the length TV = a and TR = b and RE = a, EH=b, due to symmetry, it also follows on the opposite side. HS = UW = a and US = b. Let us assume, TX = c and which is same as VX, same holds for other cases because of symmetry. Because of these the angle VTX and XVT are same and is 45°. It is presumed that angle TXV is 90°.

From the Fig. 4, it is seen that $\frac{a}{b} = \varphi \Rightarrow a = b\varphi$. From the trigonometry length c is given by,

$$c = b\varphi \sin 45^\circ$$
$$\therefore c = \frac{b\varphi}{\sqrt{2}} \tag{4}$$

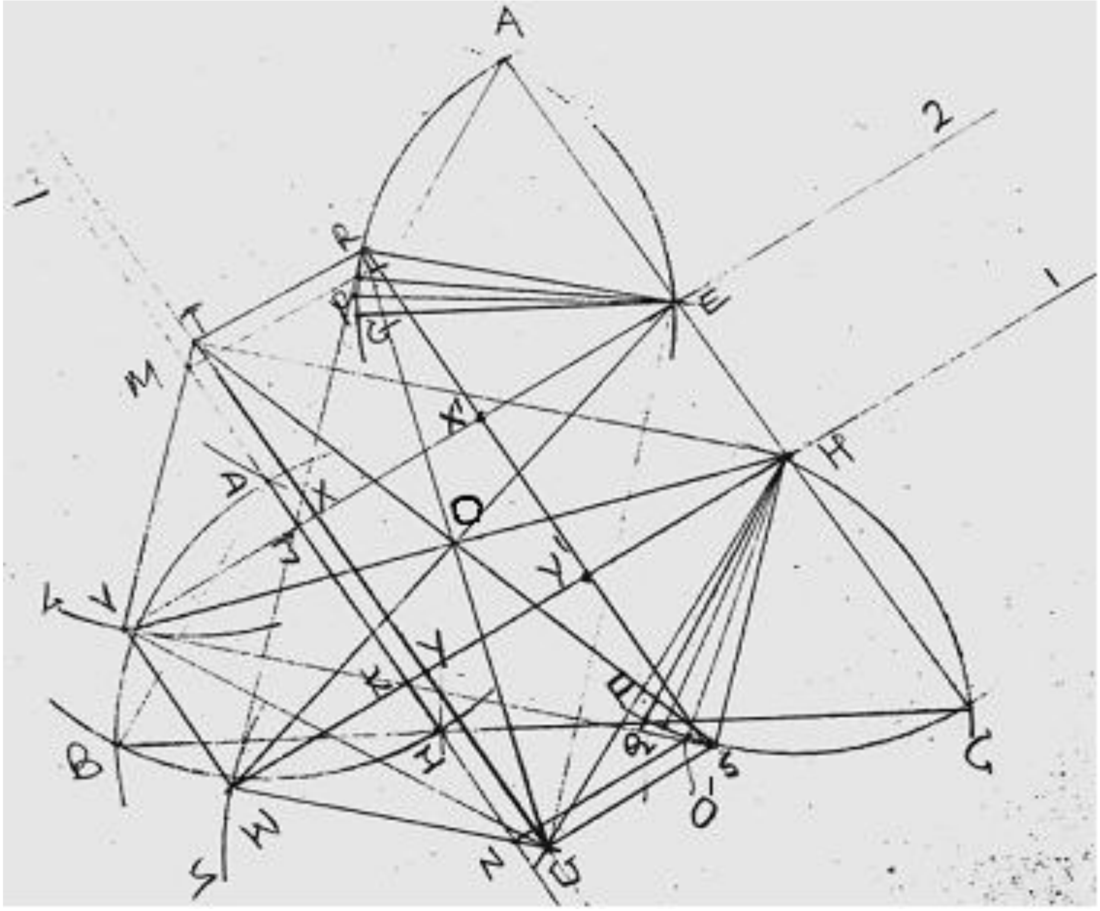

Figure 4: Properties of golden section octagon

Considering the trapezoid VTRE and WUOH, the area of these two trapezoids can be easily calculated by adding the area of two right angle triangles TXV and REX’ and a rectangle TRXX’. One more area of the rectangle VEHW to get the area of the complete golden octagon, given as follows.

Total area of the golden Octagon

$$A = 2\left[\left(\frac{\varphi b}{\sqrt{2}}\right)^2 + \left(\frac{b^2\varphi}{\sqrt{2}}\right)\right] + \left[\frac{2\varphi b}{\sqrt{2}} + b\right]b \tag{5}$$

The geometry of newly constructed golden section octagon without the construction line is seen in Fig. 5.

Conclusion

Systematic construction procedure for constructing the geometry for golden section octagon is presented in this article. The geometry is created using the concept of golden section and golden ratio in equilateral triangle. The exterior angle of the created geometry is 135° which proves and defines the shape of the octagon. The mathematical and geometrical properties are observed in the constructed golden section octagon. Golden section octagon design presented in this article is useful for engineers, artists, architects and mathematicians for drawing the various designs. The geometrical construction method presented here is applicable at any scale and valid for any construction and geometrical parameters.

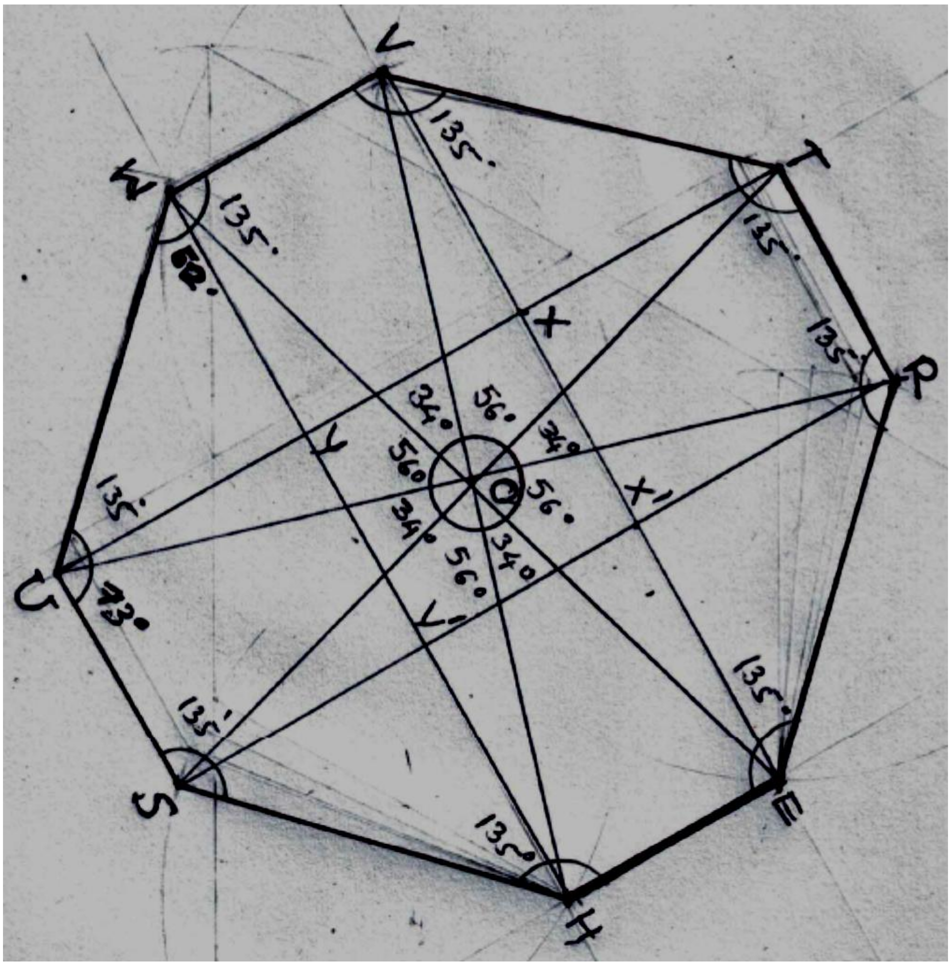

Figure 5: Golden Section Octagon

Appendix I – Procedure to obtain golden section on line

To obtain the point on the line AB such as the division of the longer line to shorter lines is in golden section, the procedure is given as follows.

Draw square of any size from the corner B on line of any length, assuming point A is not available. This line is 60^o to the horizontal line starting from corner B. Name this square as $BB_1B_2B_3$ as shown in Fig.6. Now divide the line B B_1 into half. Take the point at half a distance named B_4. Join $B_4 – B_2$ at right of the corner. Draw an arc from B_2 by taking length $B_4 – B_2$ from keeping compass point on B_4.

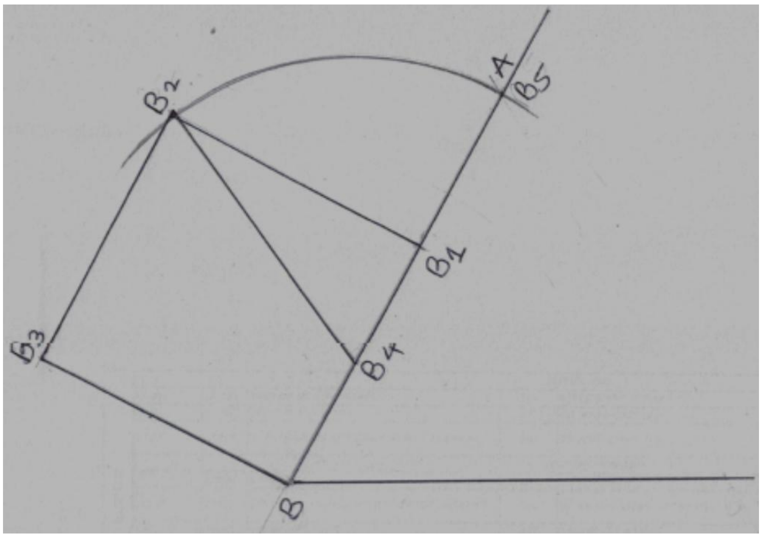

Figure 6: Procedure to obtain golden section on line

By doing this, it gives another point on inclined line going from B, named this new point B_5. We obtain, BB_1/B_1B_5 = 1.6180 and BB_5/BB_1= 1.6180. With this procedure we obtain the two divisions of lines which are in golden ratio. New point B_5 is the same as A in Fig. 1. The same procedure can be repeated for all other sides and the corners.

Another way to obtain the points which will divide lines in golden section is to simply measure the numbers following Fibonacci sequences. Such as if line is of 8 mm, take division of 5, and 3 on that line to obtain golden section. If total length is 13 mm, obtain division of 8 mm and 5 mm on the line to obtain the golden section, same is repeated from all corners and sides to achieve this ratio of division of line approximately equal to 1.6180.

Acknowledgement

Author would like to thank the reviewer for their constructive and helpful suggestions for improving the structure of the manuscript.

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