

# **Cognitivist Lesson Plans: A Tool for Effective Teaching for Mathematics Teachers**

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## **Abstract**

Effective teaching of School mathematics has always been an area of concern. Although the B.Ed. curriculum includes sufficient elements in order to train mathematics teachers, but most of the models, methods and approaches that are taught, are done as separate entities and lack a complete, connected, logically arranged mechanism to integrate subject and content-specific requirements with mathematical thinking skills. An Action research is conducted to resolve the above stated problem with B.Ed. student-teachers (who have opted for Mathematics subject as an elective) of the Navrachana University. The solution is presented as a ‘Cognitivist Lesson Plan’ format, which is executed and the result is then reflected upon for further action or modification.

## **Key words**

Mathematics Education, Mathematical thinking skills, Cognitivist Lesson Plans

## **Introduction**

The present paper is a report of an Action research conducted by the Author, who is a mathematics teacher-educator in the B.Ed. program of the Navrachana University, Vadodara, Gujarat, India. ‘Mathematics Pedagogy’ is one of the courses that is offered to the Science and the Math graduates and post graduates enrolled in the B.Ed. program. One of the important contents of this course is ‘Preparing Lesson plans’ on topics of school mathematics. In order to bring about specific changes in this area, so that student-teachers gain more efficiency in preparing Lesson plans that fulfill the subject requirement as well as they understand and integrate mechanisms to

promote mathematical thinking through their Lessons, the Author followed the following steps that align to an Action research.

“Action research is done in order to create knowledge, which is based on explorations done within specific and often practical settings. The drive of action research is to help the researcher learn through action conducted, that can lead further development of knowledge in the target area”<sup>1</sup>. “Reason and Bradbury (2006) describe action research as an approach which is used in designing studies which seek both to inform and influence practice”<sup>2</sup>. Sagor (2000)<sup>1</sup> in his book on Action Research, provided seven steps to be followed to conduct an Action research. The Study further is reported in accordance to those steps.

### **Step 1: Selecting a Focus**

The need to focus on thinking have never been so urgent or predominantly looked for until the present era. Education systems throughout the world have been striving to refurbish this need, which is apparent from their policies, frameworks, curricula<sup>3,4,5, 6,7,8</sup>. Jean Piaget and many other prominent psychologists like Erickson, Bruner, and Berlyne adhere to the fact that rightful thinking need to be taught by presenting appropriate experiences. The different subjects taught in schools thus are created to deliberate this goal.

Mathematics as a subject enjoys supremacy with respect to its ingrained ability to initiate, involve and award its seekers with analytical, critical and creative thinking. Mathematics educators get an added advantage on this, with myriad scopes overtly and covertly integrated within the contents they teach. In spite of this advantage, there are very few mathematics teachers who are able to develop mathematical thinking among their students. This is due to their strong adherence to the transaction of procedural knowledge needed to solve a mathematical problem in a fixed, defined context. Mathematical thinking, in particular, means not just solving math problems but to see the number, or the symbol, formula, equation, or the statement with its realistic meaning; to understand the underlying relationships, see the patterns and make conjectures; use varied mathematical and established facts to make new conjectures.

Thus the major focus of the Author was to understand the concept of ‘Thinking’ in more depth; relating the same to the Mathematics subject –related requirements and presenting that as a proper structure to the student-teachers.

### **Step 2: Clarifying Theories**

There are many learning theories, taxonomies and frameworks guiding educators to identify, design, and implement instructions that aim to develop mathematical thinking among students. In the present paper, the author studied the ‘thinking processes and skills’<sup>9, 10</sup> and then attempted to integrate the learnings to develop Lesson Plans for mathematics teaching, terming the same as ‘Cognitivist Lesson Plans’.

- **Concept of ‘Thinking’ with respect to Bloom’s Revised Taxonomy and Marzano et al.**

Both the stated Studies have explained ‘thinking’ in terms of ‘thinking processes’ and ‘thinking skills’. The same is tabulated below and explained in paragraphs ahead.

Cognitive Processes	Sub Categories	Types of Knowledge used in cognition
Remember	Recognizing, recalling	<i>Factual Knowledge</i> <i>Conceptual Knowledge</i> <i>Procedural Knowledge</i> <i>Metacognitive Knowledge</i>
Understand	Interpreting, exemplifying, classifying, summarizing, inferring, comparing, explaining	
Apply	Executing, implementing	
Analyze	Differentiating, organizing, attributing	
Evaluate	Checking, critiquing	
Create	Generating, planning, producing	

**Table 1: Bloom’s Revised Taxonomy<sup>9</sup>**

Thinking Processes (p 32)	Thinking Skills (p 68)
<i>Concept formation</i>	Focusing skills (defining problem, setting goals)
<i>Principle formation</i>	Remembering (encoding, recalling)
<i>Comprehension</i>	Organizing (comparing, classifying, ordering, representing)
<i>Problem Solving</i>	Analyzing (identifying components, relationships & patterns, main ideas and errors)
<i>Decision Making</i>	Generating (inferring)
<i>Research</i>	Integrating (consolidating)
<i>Composition</i>	Evaluating (establishing criteria, verifying)

**Table 2: Marzano et al. ‘Thinking Processes & Skills’<sup>10</sup>**

Bloom’s “Cognitive processes with its Sub-categories” and Marzano’s “Thinking skills” as shown in Table 1 and 2, can be considered to be micro skills or “simpler cognitive operations” required in the development of Marzano’s “Thinking processes” as in Table 2 and Bloom’s “Knowledge types” as in Table1. The “Thinking skills and the Cognitive processes” can be

considered as the means or tools used to achieve the outcome or product termed as “*Thinking processes*” by Marzano et. al and “*Knowledge types*” by Bloom.

Thus, while planning a Lesson, if the “*Thinking processes*” or the “*Knowledge types*” are considered as the ‘learning outcomes’ of the Lesson, then it would be easier for teachers to engage their students with the above referred “Thinking skills” during the instructional as well as the assessment phase. So, in order to design mathematical instruction that aims to promote mathematical thinking among students, some of the “*Thinking processes*” and the “*Knowledge types*” as enlisted in Table 1 and 2 have been used to create a flexible framework for teaching mathematics in the present Paper ahead. In doing so, the specific subject-related requirement is also needed to be taken care of.

To design effective instruction in mathematics, educators and policy makers emphasize on the transaction of ‘conceptual knowledge’ and associated ‘procedural knowledge’ for effective problem solving. Conceptual knowledge of a content enables problem solvers to use higher cognitive skills and apply appropriate or discover new procedures to solve the mathematical problem. “The National Research Council and NCTM declared a student to be mathematically proficient, if he possesses: Conceptual understanding, Procedural fluency, Strategic competence, Adaptive reasoning, and Productive disposition”<sup>11</sup>.

Thus, keeping the above requirements for effective teaching-learning in mathematics, let us focus on the concept of Lesson Planning.

- ***Lesson Planning in Mathematics***

“L.B. Stands defines a Lesson plan as ‘Plan of Action’ executed by a teacher to teach a specific topic”<sup>12</sup>. Depending upon the requirement of the topic to be taught, teacher uses different approaches to teach, which is recorded in specific formats for unanimity. One of the most prevalent type of Lesson plan format that is followed in the teacher community and is also a part of the B.Ed. curriculum is the Herbartian Lesson Plans which includes five steps besides the General and the Specific Objectives. They are “1. Introduction, 2. Presentation, 3. Comparison and Association, 4. Generalization, 5. Application”<sup>13</sup>. These plans support traditional form of teacher centred teaching and is proved not to be much effective in developing thinking skills in students. Lesson plan that support an extreme opposite, i.e. student centred approach of teaching, widespread in education is the Constructivist approach. In this, teacher provides learning experiences to students, in form of activities, manipulatives, case studies, etc. and then uses scaffolding to guide the students to

discover the learning on their own<sup>14</sup>. There are different formats and models that are used to make Constructivist lesson plans.

Different methods of teaching mathematics like the Analytical and synthetic method; Inductive and deductive method; Heuristic method, Problem-solving method, Project method are taught to the B.Ed. students in Navrachana University, Vadodara along with the Herbartian and the Constructivism Lesson plans. But each are taught as separate entities; what was required was a common Lesson plan format that fulfills the requirement of effective mathematics teaching - ingrained with opportunities to develop mathematical thinking in students and includes the positives of all the above mentioned approaches and methods.

In order to fulfill this requirement, the Author integrated the required approaches of teaching mathematics along with the thinking processes and skills as indicated by Bloom and Marzano et al., and created a Lesson Plan format and termed that as Cognitivist Lesson Plans.

- ***Cognitivist Lesson Plan Format***

Learning outcomes for a specific sub-topic in mathematics is expected in the following 5 areas:

***1. Transaction of the Math Content needs to be done to deliver the (Conceptual Knowledge in realistic and mathematical forms):***

- Conceptual Meaning: Realistic understanding of the Math content
- Mathematical Reasoning: Connecting the realistic understanding to the mathematical understanding with proper reasoning
- Mathematical Representation: Clearly stating or consolidating the mathematical formula or definition or characteristics of the target sub-topic

***2. Related Computations need to be clearly explained in terms of the (Procedural knowledge):***

- Algorithmic Processes: Explanation of the steps required to solve a mathematical problem
- Variable Methods: Showing or allowing students to explore different theoretical and mental methods to find the solution
- Calculations: Clear explanation of the required calculations and the probable errors while applying different mathematical operations

***3. Applications of the Concept and the Computations need to be in the form of:***

Simple Problems with known contexts

- Interpretation of mathematical language
- Conversion to mathematical representation

- Computation
- Verification

#### **4. Advanced Applications:**

Problems with unknown contexts or challenging situations related to the specific content

#### **5. Through-out the transaction, development of the following Cognitive skills:**

- Basic Cognitive skills like focusing, recalling, organizing, representing, computing, visualizing, estimating, generalizing etc. –*Transacted while executing instructions to achieve Learning outcomes 1 and 2.*
- Higher Cognitive skills like analyzing, synthesizing, evaluating, creating – *Transacted while executing instructions to achieve Learning outcomes 3 and 4.*

### **Step 3: Identifying Research Questions**

Development of the Cognitivist Lesson plan was not enough, its effectivity, in terms of the ability of teachers to make such Plans, was needed to be checked. Thus, the Action research was conducted to check out :

1. The extent to which Cognitivist Lesson planning is understood by the pre-service mathematics student-teachers.
2. The extent to which mathematics student-teachers are able to make the Cognitivist Lesson plans.

### **Step 4: Collecting Data**

The data was collected from a sample of seventeen First Year (second semester) B.Ed. student-teachers of the 2019-21 batch of the Navrachana University, Vadodara, Gujarat, India. These students have ‘Mathematics Method’ as one of their elective subjects for the first three semesters in the B.Ed. program. Five of the student-teachers had Mathematics as their major subject in graduation/post-graduation; while the remaining twelve students had Science (Biology, Physics and Chemistry) as their major subject.

The author developed five sample Lesson Plans on the topic ‘Percentage’ and implemented those lessons on the Sample student-teachers. She took seven sessions of 1 hour each to demonstrate the Lessons and teach the ‘cognitivist format’ to the sample student-teachers. Students-teachers were then given out a few math topics and a time of two weeks to make detailed

‘Cognitivist lesson Plans’ for one topic. The submissions made were then evaluated with the help of the Rubric designed by the Author (attached in Appendix).

### Step 5: Analyzing Data

The detailed Cognitivist lesson plans of all seventeen student-teachers were scrutinized with respect to the Expected Learning Outcomes (as in Table 3) using two ranges – Supreme to Moderate and Minimum to Absent. The ranges indicate the quality of the ‘Instructional Processes’ used in each Lesson Plan. Points accordingly was awarded in terms of Supreme – 3; Moderate – 2; Minimum – 1; and Absent – 0. Then number of students falling in the first range (Supreme-Moderate) and second range (Minimum-Absent) was counted and converted into percentage. These details are tabulated in the Table 3.

Learning Outcomes	Instructional Processes to be used provide	% of student-teachers made Lesson Plans of the quality in the Range of-	
		Supreme-Moderate	Minimum-Absent
1. Content transaction	Conceptual Meaning	53%	47%
	Mathematical Reasoning	41%	59%
	Mathematical Representation	65%	35%
2. Computations	Algorithmic Process	53%	47%
	Variable Methods	41%	59%
	Calculations	35%	65%
3. Simple Applications	Appropriate Problems	47%	53%
	Explanation	24%	76%
4. Advanced Applications	Appropriate	23%	77%
5. Core Thinking Skills	Basic	65%	35%
	Higher	18%	82%

**Table 3: Quantum of student-teachers and the quality of ‘Cognitivist lesson Plans’**

### Step 6: Reporting Results

The outcome of the Treatment can be interpreted as follows:

#### *Learning Outcome 1 – Content Transaction*

Around 53% of the student-teachers could plan the content transaction by conveying the realistic concept or the conceptual meaning of the math topic quite satisfactorily, that is in the range of supreme-moderate.

Around 41% of the student-teachers could provide appropriate explanations or mathematical reasoning to help students make the connection between the 'concept' to its mathematical representation.

Around 65% of the student-teachers could clearly consolidate or state the mathematical representation of the math fact or content being transacted.

### ***Learning Outcome 2 – Computations***

Around 53% of the student-teachers could satisfactorily show all the algorithmic processes involved in the content to be transacted, and thus lie in the range of supreme-moderate.

Around 41% of the student-teachers showed the variable methods that can be used to reach solutions either in algorithms or in calculations.

Around 35% of the student-teachers thought it necessary to give proper explanations for the calculations involved while solving numerical problems.

### ***Learning Outcome 3– Simple Applications***

Around 47% of the student-teachers felt the need to assess the concept formation and the associated math computations with help of simple applications like direct word problems or contextual real life problems. They enlisted appropriate problems that would help strengthen the math content being taught in their Lesson Plans.

Around 24% of the student-teachers provided appropriate explanations of the Simple Problems enumerated in the Lesson Plans.

### ***Learning Outcome 4 - Advanced Applications***

Around 23% of the student-teachers included appropriate complex or higher level problems related to the math content in their Lesson Plans.

### ***Learning Outcome5 - Core Thinking Skills***

Around 65% of the student-teachers integrated instructions in the supreme-moderate range, that allowed students to exercise basic thinking skills like focusing, recalling, organizing, estimating, visualizing, verifying, generalizing in their Lesson Plans.

Around 18% of the student-teachers could provide opportunities to students to analyze, synthesize, evaluate, create in their Lesson Plans satisfactorily.

The above analysis is explained through a Sample student-teacher's LP in Appendix 2.

### **Step 7: Taking informed actions**

The analyzed data proves that more than fifty per cent of the student-teachers have understood and have satisfactorily prepared the Cognitivist Lesson Plans on different Math topics of the quality that can be termed as Supreme to Moderate.

Holistically, if we view the results, as well as while observing the student-teachers working with the Lesson plans, a lot of improved actions become imperative, which are as follows;

- Student-teachers have developed a proper understanding about conceptual knowledge and procedural knowledge and are at ease in delivering each for respective known and explored Math topics, but they mainly struggle to make the connections between both. They struggle to integrate mathematical reasoning while presenting the conceptual part and converting that to respective mathematical representation.
- Separate sessions highlighting on -(a) identification of probable errors done by school students in different Math topics (b) use of variable methods to solve a math problem (c) mental math strategies - have to be taken up to fill up the discrepancy here.
- In case of Application related section, where student-teachers had to list out appropriate word problems related to the math topic being taught and also pen down their explanations in detail. The variety, the ascending difficulty level and the explanations mattered. Many student-teachers did not provide explanations. There were a few student-teachers who completely missed out the advanced level problems or higher level tasks in their Lesson plans. Thus, the final score came out less. It was due to not lack of efficiency but may be lack of will.
- Student-teachers got a proper idea regarding the different Thinking Skills but struggled to consciously integrate the same in their Lessons.
- More rigor and time is needed in the transaction of this methodology by the teacher-educator and more practice, ability and desire is needed in the part of the student-teacher for better results.
- This format will be more applicable to in-service mathematics teachers, who have a better understanding of the subject, student abilities, curricular limitations and much stronger foundation in school mathematics.

## Conclusion

It is a well promoted fact that constructivist methodologies align very well with Thinking Skills; while Traditional modes of teaching hardly cater to the same. Cognitivist Lesson plans includes the advantage of both can be a better alternative to practice in mathematics classrooms. Teaching of the same at the B.Ed. level and allowing practice of the same during Internships would help create a bunch of mathematics teachers who could disseminate good quality mathematics teaching, especially in Indian classrooms with more density where constructivist pedagogies cannot work well. These Cognitivist Lesson plans can also be converted into Self-Learning materials and be used for online teaching.

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**APPENDIX 1****1. Rubric to check the effective use of the Cognitivist Lesson Plan Format**

Sr. No.	1. Content transaction			2. Computations			Direct Problems		Advanced Problems	Cognitive Skills	
	Concept (3 – 0)	Math Reasoning (3 – 0)	Math Representation (3 – 0)	Algorithms (3 – 0)	Variable Methods (3 – 0)	Calculations (3 – 0)	Appropriate (3 – 0)	Explanation (3 – 0)	(3 – 0)	Basic (3–0)	Higher (3 – 0)
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											
11											
12											
13											
14											
15											
16											
17											

**APPENDIX 2****COGNITIVIST LESSON PLAN MADE BY STUDENT-TEACHER 1 (LP 1)****Topic: Profit % and Loss % Class: VII****Main Teaching Points:**

1. Content transaction using realistic conceptual understanding, mathematical reasoning and representations
2. Mental calculations
3. Procedural calculations
4. Simple application of computations

5. Simple problem solving
6. Advanced level problem solving

### Teaching Plan for Main Teaching Point 1:

- **Previous Knowledge:** Students already know the concept Cost Price, Selling Price, Profit, Loss and the relationship among them. The concept of Percentage and Unitary method.
- **Media/Material:** Black board, class set- up with desks and benches, currency notes prepared by students
- **Teaching Method:** Heuristic method
- **Teaching approach:** Inductive-Deductive
- **Specific Objective:**  
Students will be able to explain the realistic concept Profit % and Loss %.  
Students will be able to establish the mathematical reasoning behind calculation for Profit% and Loss%  
Students will be able to induce the formula of Profit% and Loss%.

### Teacher & Student Activity:

1. **Development of the Realistic Concept, use of mathematical reasoning and emphasizing the mathematical language**

*[Previous day, five students were instructed by the teacher to make currency notes of Rs. 10, Rs. 50 and Rs. 100 (at least 5 of each) and to bring a specific number of day-to-day items with tags of 10, 50 and 100 on each item. A script was given to the five students (Rahul and Disha act as Retailers and the rest as Customers)]*

**T:** Good Morning students!

**S:** Good Morning Madam!

**T:** Follow my instructions to learn from the Role Play that will be conducted by us today. You can see three counters set up in front of the class. Counter 1 is tagged as 'Wholesaler', which is my counter. Rahul is the owner of the Counter 2; Disha is the owner of Counter 3. Rahul and Disha, both are 'Retailers' and Amit, Radha and Puja are 'Customers' who can buy items from the Retailers.

*[Focusing skill]*

Observe and note down the transactions of money that is happening between us during the entire Role Play. You will be asked a set of questions ahead.

**Role Play:**

Rahul comes to the Teacher and buys some items worth Rs. 2000 from the Teacher. Then Disha comes and buys some items from of the Teacher worth Rs. 1000. Before arranging the items on their Counters, both Rahul and Disha changes the price tags on each items they got. Every Rs. 10 tag is replaced by Rs. 20 tag; Rs. 50 tag by Rs. 70 and every Rs. 100 tag by Rs. 130 tag.

Now Amit comes to Rahul and buys some items for which he is charged Rs. 800; Radha buys some items and had to pay Rs. 1000 and Puja did a purchase of Rs. 700.

Teacher instructs the students to record the total purchase done from Rahul.

In the next scene, Amit, Radha and Puja purchase items of Rs. 500 each.

The Role Play ends here.

***[Visualizing skill]***

***Teacher goes ahead with a Question-Answer session, where mathematical reasoning is used to relate the realistic concept with mathematical terms/language***

**T:** In the first transaction between Wholesaler and Retailer, how what was the money transaction done by Rahul and Disha from the Wholesaler? And mathematically, which specific financial term will be used for the purchase done by Rahul and Disha?

**S:** Rahul buys items of Rs. 2000 and Disha buys items of Rs. 1000.

**T:** Since, they buy the items what terms will be used for the purchase done by them?

**S:**Cost Price

***[Recalling, Representing skill]***

**T:** Yes, very good. So for Rahul, the CP is Rs. 2000 and for Disha, CP is Rs. 1000.

**T:** What was the intention behind changing the price tags before selling their items to Customers?

**S:** Earning profit.

**T:** The Customers in Rahul's shop made a total purchase of how much?

**S:** Rs.  $(800 + 1000 + 700) = \text{Rs. } 2500$

**T:** So, with respect to Rahul, the Retailer, what statement can be made for this transaction?

**S:** Rahul sold goods worth for Rs. 2500.

**T:** Very good. So, what should be the mathematical term used instead of the above statement for Rahul's sale?

**S:** For Rahul, the Selling Price or SP is Rs. 2,500.

**T:** And when you apply the same for Disha, what is it?

***[Organizing skill]***

**S:** For Disha, the Selling Price or SP is Rs. 1,500.

**T:** Now check with Rahul's CP and SP, and also of Disha's CP and SP. What gets revealed?

**S:** Both earn the same Profit of Rs. 500.

**T:** Very Good! But, what do you think, both will be equally happy with their dealings? Although the Profit amount is same, do they have the equal stand, when their CPs are not same?

**S:** We do not know madam.

**T:** Like in case of your marks, let me give an example – You have scored 20 out of 30 in Hindi and 40 out of 50 in English. What do you do to find out your performance was actually good in which subject?

**S:** We find percentage.

**T:** Right, the same can be done in Rahul's and Disha's case to check their performance.

## **2. Use of Mathematical reasoning to establish the Formula or the Mathematical representation**

**T:** So, we need to find the percentage of what, to check Rahul's performance?

**S:** Profit

**T:** How do we find the percentage of Profit Rahul made?

**S:** We don't know.

**T:** Use the same Marks example. You can try the Unitary method, where you have to calculate your marks out of 100.

**S:** For Hindi, out of 30, I get 20....so out of 100, I get?

$$\% \text{ of marks in Hindi} = \frac{20}{30} \times 100 = 66.6\%$$

*[Computing skill]*

**T:** Use the same reasoning in case of Rahul to find his Profit %.

**S:** His profit is Rs. 500, but out of how much?

**T:** Of course, he can earn only if he makes a proper investment, how much was that?

**S:** His CP, because he invested that Rs. 2000.

So, it will be- for Rs. 2000, he makes a profit of Rs. 500; then for Rs. 100, his profit will be?

$$\text{Rahul's Profit \%} = \frac{500}{2000} \times 100 = 25\%$$

**T:** Find the same for Disha.

$$\text{S: Disha's Profit \%} = \frac{500}{1000} \times 100 = 50\%$$

**T:** So what can you say now, on Rahul and Disha's status?

**S:** Disha's Profit % is more than Rahul.

**T:** Exactly, hope it is clear now that, only calculation of Profit, does not give us a real picture when we want to make comparisons. Companies need to compare their profits made with respect to previous years. Since, the initial CP or investment may not be same every year, the % profit or loss can give a better picture. There can be many such examples.

**T:** Now, see the calculation you did in Rahul and Disha's case using unitary method to calculate profit %. Can you come down with a Formula which can be directly used, when we know the initial investment or the CP and the Profit/Loss (or SP from which it can be calculated)?

$$\text{S: Profit \%} = \frac{\text{Profit}}{\text{CP}} \times 100$$

$$\text{Loss\%} = \frac{\text{Loss}}{\text{CP}} \times 100$$

*[Generalizing skill]*

#### Author's Feedback for Learning Outcome 1:

1. Conceptual meaning – the realistic understanding has been transacted impeccably. (Scored 3)
2. Mathematical Reasoning – Also integrated throughout excellently without any flaw (Scored 3)
3. Mathematical Representation – mathematical language well established and the final formula also induced well. (Scored 3)

Basic thinking skills: focusing, recalling, organizing, visualizing, verifying, generalizing skills have been taken care of. (Scored 3)

Calculation of Numerical: —

Tr → Calculate Profit % or Loss % for the following —

i) C.P = 590 S.P = 649 ii) C.P = 800 S.P = 760

St → i) Profit = 649 - 590 = 59  
 $\therefore$  Profit% =  $\frac{59}{590} \times 100 = 10\%$

ii) Loss = 800 - 760 = 40  
 $\therefore$  Loss% =  $\frac{40}{800} \times 100 = 5\%$

#### Author's feedback for Learning Outcome 2:

1. Algorithms – use of formula could be shown explicitly. (Scored 1)
2. Variable methods – Although not explicitly shown here, the application of unitary method to calculate % is shown in 1<sup>st</sup> segment. (Scored 2)
3. Calculations – decimal & fractional forms missing. (Scored 1)

Teacher & Student Activity	Skill used
<u>Similar looking Problems</u> —	
<u>Set-I:</u> —	
(i) Shivanshi bought a sari for ₹ 2200 & sold it for ₹ 2420. Was there a profit/loss & what percent?	Comparison
(ii) Shivanshi bought a sari for ₹ 2200 & sold it at 10% profit. What was the selling price of sari?	
<u>Set-II:</u> —	
(i) Arnav bought a wrist watch for ₹ 900 & sold it at ₹ 810. Find the profit/loss percentage.	
(ii) By selling a wrist watch at ₹ 810 Arnav faced loss of 10%. Find its cost price.	

### Author's feedback for Learning Outcome 3:

1. Appropriateness of Simple Problems— The problems given are appropriate, fulfills the purpose of seeing relationships between the taught concept and simple real life applications. Also similar looking problems that require the use of higher focusing and analyzing skills are included (Scored 3)
2. Explanation – Not provided. (Scored 0)

Teacher & Student Activity	Skill used
<u>Advance Application:</u> —	
(i) A shopkeeper bought 120 sheets at ₹ 20 each. In his house 18 were consumed & he sold the remaining at ₹ 30 each. Find his profit/loss %.	Mathematical Reasoning
(ii) By selling a sofa for ₹ 10,240 Ronnie incurred a loss of 20%. If he wanted to make a profit of 6% at which price should he have sold it?	

### Author's feedback for Learning Outcome 4 and 5:

1. Appropriateness of Advanced Problems— The problems given are appropriate. They will require the higher thinking skills of analyzing, synthesizing and evaluating (Scored 3)
2. A creative task could have been included for exercising the thinking skill of creating (Scored 0)