A Perspective on Geometric Pedagogy for Secondary School Students in a Virtual Classroom

Radha Madhavi Duggaraju* and Lipika Mazumdar

School of Science, Navrachana University, Vasna-Bhayli Road, Vadodara- 391410, Gujarat, India

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*Corresponding Author: rmadhavi17@gmail.com

Abstract

We were encouraged to write this article after observing closely a group of students in standard IX, taking the online lessons in Geometry, due to the closure of schools in the Covid-19 pandemic. It was evident from their body language that the diligent efforts of the teacher were not sufficient to grab their attention. Diving deeper, we found that the topics in Geometry involved construction, which relied heavily on explaining the steps by doing them physically in the classroom. To address this and similar issues, we have proposed a pedagogy with an objective to create an interactive environment in virtual geometry classes, where the mathematical concepts and the steps of construction are taught with the traditional methods blended with modern graphic software tools.

Keywords

Geometry, pedagogy, interactive environment

Introduction

Presently many schools across the world are closed as a precautionary measure against the spread of Covid-19 pandemic. The age-old method of classroom teaching has been subsequently shifted to online education, which is a new way of teaching-learning for the instructors as well as the students. Many school children are seriously affected by this sudden shift, whereas their instructors are struggling to find innovative methods of engaging them as

effectively as in classroom teaching. But, as we all know that online education is imposing serious challenges at various levels of teaching-learning, a few of them being discussed below:

- Online teaching requires development of innovative teaching strategies and techniques. Instructors need to be tech savvy to promptly shift to online mode of teaching from the traditional classroom education.
- It is challenging for any subject instructor to grab the attention of students on the computer screen and also to keep them actively engaged for the entire duration of these virtual classes. Also, different courses require different levels of engagements, and hence teaching needs to be customized as per the requirements of the subject.
- Considering the emotional state of students is also very important in online teaching, particularly during such unprecedented times, when many families are in distress due to economic, health and other issues. Children are also affected seriously due to the prolonged lockdowns and pandemic related effects on their families.
- Identifying the slow learners and giving individual attention to students who need extra help may be difficult in virtual classrooms.

Although most of the issues discussed above have evolved as effects of the ongoing pandemic and are difficult to address, but they also provide food for the thought of coming up with innovative ideas at various social, economic, or educational platforms for future use. From our perspective, the present situation also encourages us to modify our age-old methods of teaching to be blended with modern software tools, giving more freedom for experimentation to the students. Playing around with the ideas and learning by exploring is far more engaging than accepting the textbook concepts as absolute truth, as is expected in traditional classroom teachings.

In this manuscript, we present our perspective on the method of teaching geometrical concepts to secondary school students in online mode. Geometry is a science which encompasses a collection of abstract statements and proofs of these statements[1]. Studies in Geometry constitute an inexhaustible source of ideas for developing logical and creative thinking[2]. Practical problems ranging from designing a city, measuring the earth to using shadows to tell time, led to the development of this branch of mathematics[3]. Secondary school students need to have a wide variation of foundational knowledge in Geometry as the building blocks for their respective fields of choice in future. They require a more personalized approach to learn

topics like Practical Geometry, which is a blend of abstract mathematical ideas and their applications to the real world. We closely analyzed the methodologies used by teachers to deliver the topics in the areas of practical mathematics and concluded that it was really difficult to deliver both the problem-solving techniques and the abstract mathematical concepts simultaneously, particularly in online mode. In this regard, we tried to develop a blended approach to make the teaching of practical mathematics in virtual environments more effective, which can also be carried forward in regular classroom teaching later.

Drawbacks in online teaching

- As instructors are teaching remotely, necessary infrastructure like blackboards are usually not present or are difficult to install. They have to depend more on verbal communication instead of figures/graphs/drawings etc. to enhance the learning. Although power point presentations have replaced the blackboard usage, but it has been observed that it allows passive learning in students and a majority of them end up with reduced attention span in the lecture.
- 3D software for giving real time perspective of geometrical shapes is not shown or linked with the mathematical ideas. Incorporating such software requires time and training for the instructor, hence is not an easy task for them.
- There is no eye contact between instructor and student, because of the virtual classrooms. As a result, it becomes impossible to judge the engagement of the students during the class hours. The disengaged students start inappropriate usage of the electronic gadgets.

First, we discuss some preliminary suggestions which are expected to bring interactive and exploratory learning in virtual classrooms, also appropriate for teaching subjects other than mathematics.

Primary Suggestions: These suggestions can be implemented in the virtual classrooms of mathematics and can be modified as per the requirements of teaching other subjects to ensure a conducive environment for learning.

Assumption: Classroom session is of 40 minutes; Maximum classroom strength is 40.

- Putting students together in groups of 6 to 7 students enhances collaborative efforts and creates good interactive learning environment. The assignments can be designed in such a way that students need to interact with each other for solving. They are benefitted by the peer effect of learning and multiple ways of problem solving. This also makes students feel that they are not isolated in a virtual environment.
- Letting one of the groups present the contents of the previous lecture in the first few minutes of the class and giving some time to the other students to raise questions. This reinforces their learning and increases their alertness in class.
- Leading students through real world examples in mathematics and creating attractive visual effects by using different colors for different sets of parallel lines, angles or shapes gives more clarity. Integrating each lesson with short videos on topics like construction of angles or quadrilateral help the students to move along and continue the construction activity. Instructors may share these videos with students through e-mail for their later reference.
- Dynamic geometry software tools allow for exploration in geometric studies. For example, a teacher can ask students to explore how a particular quadrilateral behaves when one of its vertices is dragged and this question is one that cannot be posed in a paper-pencil environment[4]. In fact, playing around with the dimensions, rotation, magnification, etc. of the shapes and figures are possible while using these software tools, which is very important in conceptual understanding of the subject.

Some of the free and easily available software tools to teach Geometry are Geo Gebra, Geometry pad, Shapes 3D, Desmos etc. Instructors need to download and install these software tools from the internet on their computers or smart phones.

We give the following sample problem containing construction work. We also give screenshots which demonstrate the geometrical construction work using Desmos software geometry tool. This software tool is great for teaching geometry and offers a graphing calculator that can be used by students. It allows the instructors to monitor and share student work. It is even accessible for visually impaired students.

This Sample Problem is selected from the topic ‘Number Systems’ whose solution contains geometrical construction work. The abstractness of the dense packing of rational and irrational numbers in the real number system is very difficult to visualize, particularly for a student who

has just begin to learn mathematics as a concept rather than a tool to manipulate real world observations. Constructing the beautiful spiral of circles with irrational numbers as radii and their intersection with the real line gives the idea that the irrational numbers are densely packed within the real line. To begin with we give a brief introduction to the real number system.

Mathematically, a number is called a rational number, if it can be written in the form $\frac{p}{q}$, where p and q are integers and q≠0 and is called an irrational number if it cannot be written in the form $\frac{p}{q}$. Now in real sense, the rational numbers include whole numbers, terminating and recurring decimal numbers whereas the irrational numbers are non-terminating and non-recurring decimal numbers, for example 1.010010001…is an irrational number. The collection of all rational numbers and irrational numbers together make up what we call the real numbers, denoted by R. Therefore, a real number is either rational or irrational. Also, every real number is represented by a unique point on the number line and vice versa.

Sample Problem: Show how the irrational number $\sqrt{5}$ can be represented on the number line

Solution: The following steps of construction in the solution of the above problem is shown in Figures 1 and 2.

1. On the number line, choose points A = 0 and B = 1. Now line segment AB is of unit length. Construct BC of unit length perpendicular to AB. Join the points A and C to get a right triangle ABC. Using the Pythagoras theorem, we see that AC = $\sqrt{1^2 + 1^2}$= $\sqrt{2}$.
2. Construct CD of unit length perpendicular to AC. Join A and D to get a right triangle ACD. Now using Pythagoras theorem, the length of hypotenuse, AD= $\sqrt{\left(\sqrt{2}\right)^2 + 1^2}$ = $\sqrt{3}$
3. Again, construct DE of unit length perpendicular to AD and join AE to get a right triangle. Similarly, as in the above steps we get, AE = $\sqrt{(\sqrt{3})^2 + 1^2}$ = $\sqrt{4}$. Continuing the similar procedure by constructing EF of unit length perpendicular to AE and joining AF, we get AF = $\sqrt{(\sqrt{4})^2 + 1^2}$ = $\sqrt{5}$.

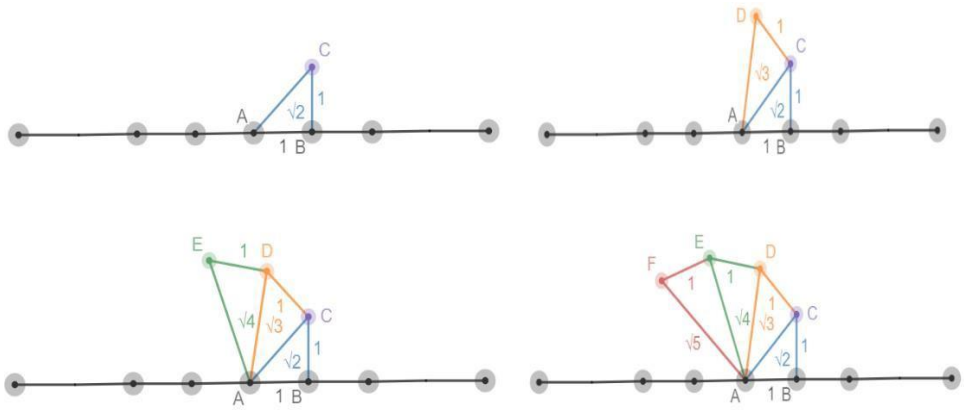

Figure 1: Steps of the solution

4. Using a compass, with centre A and radius AF, draw an arc which intersects the right side of the number line at a point which represents $\sqrt{5}$.

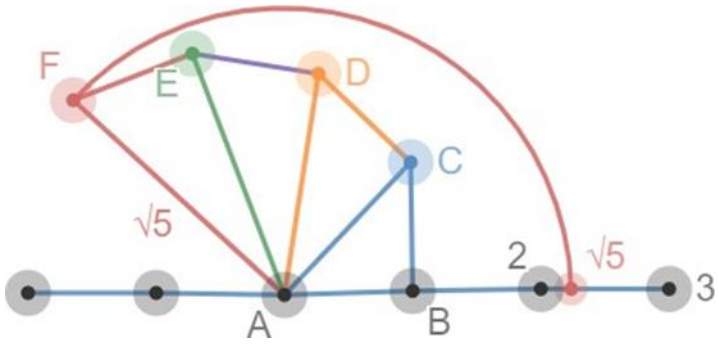

Figure 2: Representation of an irrational number $\sqrt{5}$ on number line

Square root spiral: The following Figure 3 shows a beautiful spiral depicting $\sqrt{2}, \sqrt{3}, \sqrt{4}, \ldots$ and their representation on the number line.

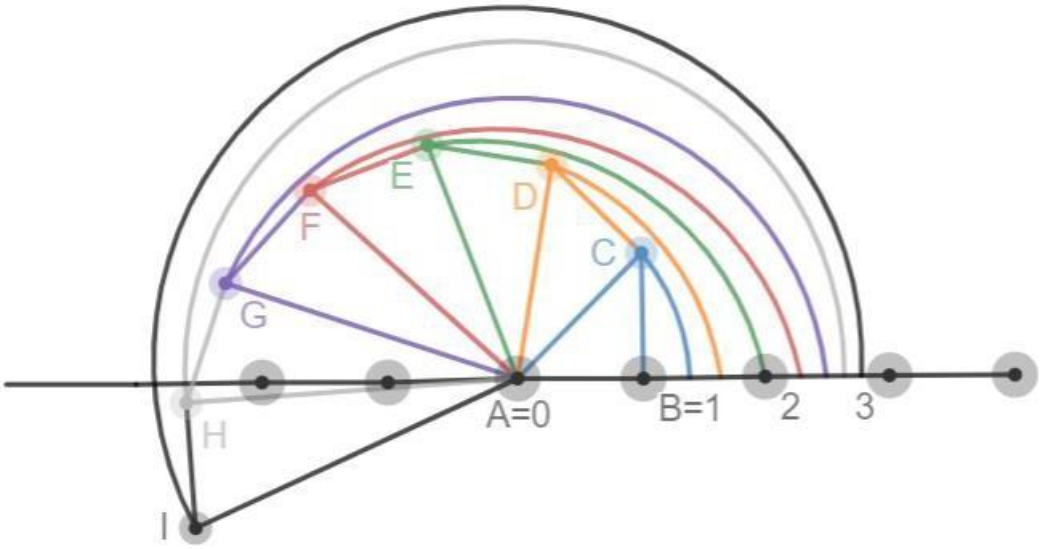

Figure 3: Square root spiral

The following table demonstrates different irrational numbers in the above Figure 3 with their respective colours of arcs on number line.

Irrational number	Color of arc
√2	Blue
√3	Orange
√4	Green
√5	Red
√6	Violet
√7	Grey
√8	Black

Table 1: Irrational Numbers and Colour of Arcs

The above construction helps the students to internalize some very important features of the number line, representing the set of real numbers R, which are discussed as below:

- Well Ordered property of the real numbers, i.e. given any two real numbers x and y, one and only one of the following is true: $(i) x < y (ii) x = y (iii) x > y$. The real line is ordered from left to right, meaning while comparing any two numbers, we know that the number on the right is bigger. Thus, for square roots, it can be seen very easily seen that $2 = \sqrt{4} > \sqrt{3} > \sqrt{2} > \sqrt{1} = 1$ [Fig.3] or we can say that if

$n > m$. we have $\sqrt{n} > \sqrt{m}$.

- Denseness of the set of rational numbers and set of irrational numbers, i.e., the numbers in the number line are not isolated points but are densely packed in it. More specifically, we can say that there is no next real number in the real line. From the definition of irrational numbers, we know that irrational numbers are non-terminating and non-recurring decimal numbers, say for example $\sqrt{2} = 1.4142135$ ..... and $\sqrt{3} = 1.73205080$ .....Now, we can find an infinite number of rational numbers between these two numbers such as 1.5, 1.51, 1.511, 1.5111,…..Similarly, between any two rational numbers, say 1 and 2, we can find an infinite or rather uncountable number of irrational numbers say $\sqrt{2}$, $\sqrt{2.1}$,$\sqrt{2.11}$,…..and so on. Visualization of this concept with the software tools helps them to understand the abstract concepts of limiting values of functions, continuity, and differentiability to a greater extent in further studies.

Hence, we see that the abstractness in the mathematics can be reduced by adding visualization with software tools in online as well as classroom teachings. This will add a great value to the mathematical understanding of the students and applying it for studying the infinitesimally small to infinitely large objects in the Universe, recognize patterns around them and many such things in their respective courses.

The following is the link for the video of the above construction which we have prepared and uploaded on YouTube: https://youtu.be/jlatb0MK_NA

Also, we discuss about another problem in Geometric constructions in Mathematics with the help of Desmos Software. The standard method of constructing the angle bisectors can be demonstrated with the help of the software, which is very easily grasped by the students, even in virtual mode of teaching. The link for the lecture is: https://youtu.be/YG6UUwqwwUs

We have made an effort to get feedback for the video lecture containing the solution of the Sample problems from a teacher Ms.Tannvi Trivedi, presently teaching Mathematics to class IX students and from a group of randomly selected class IX students. Ms. Trivedi has found this software tool to be useful for explaining the problems involving construction in Geometry.

The Feedback of a Group of Class IX students is as follows:

- Venkata Sai Sachin, Class IX: The video was excellent. I understood the concept of representation of irrational numbers and got all my doubts cleared.
- Dutt Parmar, Class IX: The video was great, and the way of teaching was also great. Now representation of irrational numbers on number line is clear to me.
- ShauryaThaplyal, Class IX: This was really a great knowledge and easier way to do a very important topic. This will really help me. Great work and thank you for the effort.
- Neel Shah, Class IX: This video gave a clear understanding of how $\sqrt{5}$ is represented on number line. Also, more videos of this kind can be very helpful.
- Parth Dighe, Class IX: I liked this video very much. Clear concept. Depiction is good. Points are explained very clearly.
- Monit, Class IX: This video is very useful as the Number systems topic is quite difficult. But with this video all my doubts about representation of irrational numbers are cleared.

Conclusion

The world as we know has changed irreversibly with Covid-19. Virtual classrooms are now the de-facto mode of teaching in schools and colleges. Without visualization of the mathematical models, students would not be able to comprehend the intricacies of the subject and learning may become mechanical with the routine problem-solving approach. Traditionally, the geometrical problems were taught using black board and wooden geometrical tools. However, in the online teaching scenario, teachers are mostly teaching remotely and do not have access to the earlier tools they were familiar with. In this situation, software technology tools can be of great help and can even be incorporated with regular classroom teaching after pandemic is over. Necessary software tools are available for instructors on internet, free of cost and are very easy to learn without any requirement of formal training.

A good fundamental knowledge of the subject at an early stage in student life will make it easy for them to understand the advanced knowledge of the related subjects as they progress.

We hope that our perspective encourages the instructors to think and work in the direction of modifying the age-old teaching methods for those branches of science which are complex and needs visualization for effective understanding of the nuances of the subject. For further readings, we suggest the following literature, which we found extremely valuable towards developing this manuscript.

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